954/2 **STPM 2006**

MATHEMATICS T (MATEMATIK T)
PAPER 2 (KERTAS 2)

Three hours (Tiga jam)

JOHOR BAHRU
(Johore Bahru)

PEPERIKSAAN PERCUBAAN STPM 2006
(STPM Trial Examination 2006)

Instruction to candidates:

Answer **all** *questions. Answers may be written in either English or Malay.*

All necessary working should be shown clearly.

Non-exact numerical answers may be given correct to three significant figures, or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Mathematical tables, a list of mathematical formulae and graph paper are provided.

Arahan kepada calon:

Jawab **semua** *soalan. Jawapan boleh ditulis dalam bahasa Inggeris atau bahasa Melayu.*

Semua kerja yang perlu hendaklah ditunjukkan dengan jelas.

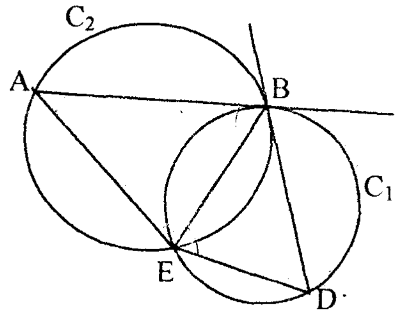
Jawapan berangka tak tepat boleh diberikan betul hingga tiga angka bererti, atau satu tempat perpuluhan dalam kes sudut dalam darjah, kecuali aras kejituan yang lain ditentukan dalam soalan.

Sifir matematik, senarai rumus matematik, dan kertas graf dibekalkan.

This question paper consists of 4 printed pages.
Kertas soalan ini terdiri daripada 4 halaman bercetak.

* This question paper is confidential until the examination is over.

1.



Given that AB is a tangent to the circle C_1 and DB is the tangent to the circle C_2, prove that $AE \bullet ED = (EB)^2$. [4 *marks*]

2. Find the solution for the differential equation $\frac{dy}{dx} = e^{2x+y}$ satisfying the boundary condition $x = 3, y = -6$ by giving your solution in the form of $y = f(x)$. [5 *marks*]

3. In a chemical process, a chemical substance is produced inside a container at a rate which, at any time, is k times the amount of the substance left in the container at that instant. The chemical substance is also removed from the container at a constant rate of α. Obtain a differential equation for the amount of the chemical substance, x in the container at the time t.

If the initial amount of chemical substance is $\frac{4\alpha}{3k}$, prove that the amount of the chemical substance at time t is given by $x = \frac{\alpha}{3k}\left(3 + e^{kt}\right)$. [6 *marks*]

With the aid of a graph, briefly describe how x changes with t. [2 *marks*]

4. By using the identities
$\sin(A + B) \equiv \sin A \cos B + \sin B \cos A$ and $\sin(A - B) \equiv \sin A \cos B - \sin B \cos A$, prove that

$$\sin P - \sin Q = 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$$ [3 *marks*]

Hence,

a. solve the equation $2\sin(\theta + 70^\circ) - 2\sin(\theta + 10^\circ) = \sqrt{3}$ for $0^\circ \le \theta \le 360^\circ$. [3 *marks*]

b. find x in the range of $0 \le x \le \frac{1}{2}\pi$ that satisfy the inequality $\sin 5x - \sin x \ge \cos 3x$. [4 *marks*]

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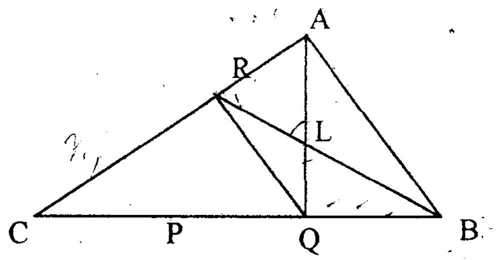
5. The velocities of 2 boats A and B are $(3\mathbf{i} - 2\mathbf{j})\text{ms}^{-1}$ and $(x\mathbf{i} + y\mathbf{j})\text{ms}^{-1}$ respectively. Initially, the position of boat B relative to boat A is $(300\mathbf{i} - 300\mathbf{j})$m. Given that the speed of B is $\sqrt{5}\text{ ms}^{-1}$ and the two boats subsequently collide,
   a. find the possible numerical values of x and y. [6 *marks*]
   b. find the possible times taken for the collision to occur. [4 *marks*]

6. Prove that triangles on the same base and between the same parallel lines have equal areas. [5 *marks*]



In the figure, BA and QR are parallel and $CP = PQ = QB$. The lines AQ and BR intersect at L.

   a. Prove that ΔRCQ is similar with ΔACB and $AR = \frac{1}{3}AC$. [4 *marks*]
   b. Prove that the area of ΔARL = the area of ΔBLQ. [2 *marks*]
   c. Deduce that the area of ΔBPR = the area of quadrilateral $ARPQ$. [2 *marks*]

7. The number of fish caught by 500 contestants in a fishing competition at a lake is given in the following table.

Number of fish	0	1	2	3	4	5	6	7 or more
Relative frequency	0.016	0.02	0.098	0.224	0.196	0.172	0.108	0.166

   a. State the mode and the median number of fish caught by the contestants. [2 *marks*]
   b. For the number of fish which is more than 6, the mean number of fish is 7.843. Find the mean number of fish caught by the contestants. [2 *marks*]

8. Soap powder is packed in packets of two different sizes. The mass in each small packet may be regarded as a normal random variable with mean 505g and standard deviation 10g while the mass in each large packet is another independent normal random variable with mean 1005g and standard deviation 20g. Find the probability that
   a. the mass of one randomly chosen large packet exceeds the total mass of 2 randomly chosen small packets. [3 *marks*]
   b. the mass of one randomly chosen large packet is less than twice the mass of one randomly chosen small packet. [3 *marks*]

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9. The continuous random variable, X, has the probability density function f, given by

$$f(x)=\begin{cases} cx & 0\le x\le 5 \\ 5c & 5<x\le 8 \\ 0 & otherwise \end{cases}$$

where c is a constant.

i. Show that $c=\frac{2}{55}$, [2 *marks*]
ii. find the cumulative distribution function of X, [3 *marks*]
iii. find the median of X. [2 *marks*]

10. Given two dice, one is biased and the other unbiased. The probability of getting a six from the biased die is $\frac{1}{3}$.

a. One die is chosen at random and rolled. Find the probability of getting a six. [2 *marks*]
b. If the roll of the die has resulted in a six, find the probability that
   i. it was the unbiased die that was rolled. [3 *marks*]
   ii. a second six is obtained when the same die is rolled again. [4 *marks*]

11. The following data shows the number of computer sets sold by a firm in a period of 28 weeks.

20	15	10	10	13	25	7
4	11	4	5	14	12	7
8	14	7	5	21	5	10
16	9	8	4	12	13	6

a. Construct a stem-plot for the above data. [2 *marks*]
b. Calculate the mean and the standard deviation of the number of computer sets sold per week. [4 *marks*]
c. Find the median and semi-inter quartile range of the above data. [3 *marks*]

12. The discrete random variable X has the probability distribution

x	<22	22	23	24	25	26	>26
$P(X=x)$	0.209	0.102	0.120	0.128	0.123	0.106	0.212

The mean value of the classes '<22' and '>26' are 19.5 and 28.4 respectively. Show that the mean and the variance of the distribution correct to one decimal place are 24.0 and 9.6 respectively. [4 *marks*]
If X has a binomial distribution, $B(n, p)$, find the values of n and p. [3 *marks*]
By using a suitable approximation, find $P(19<X\le 27)$ [4 *marks*]
If two independent observations are taken from this distribution, find the probability that exactly one of them is more than 25. [4 *marks*]

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